SIME 2026

SIMC Leadership Group

30 January 2026

Problem 1. Let $(n_i)_{i\geq 1}$ be an infinite increasing integer geometric series. If

$$\log_{67}(n_1) + \log_{67}(n_2) + \cdots + \log_{67}(n_9) = 2025,$$

how many distinct sequences $(n_i)_{i\geq 1}$ exist?

Problem 2. Raymond owns 1 long black sock, 4 short black socks, 3 long white socks, and 4 short white socks. He picks socks randomly until he finds a pair of the same sock. If the probability that the pair is of black socks can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n, find $m + n$.

Problem 3. Let N be the number of pairs of positive integers (a, b) such that $a, b \leq 2026$ such that

$$a^2 + b^2 = \gcd(a, b) + 2\operatorname{lcm}(a, b).$$

Find N mod 1000.

Problem 4. Let $a_0 = \frac{125}{81}$ and $a_n = a_{n-1}^2 - 2a_{n-1} + 2$. If $\prod_{k=0}^{\infty} a_k = \frac{m}{n}$ for relatively prime positive integers m and n, find $m + n$.

Problem 5. Let a and b be positive integers satisfying $a^{b^2} = b^a$. If A is the sum of all possible values of a, and B is the sum of all possible values of b, compute AB.

Problem 6. In triangle ABC, $\angle C = 45^\circ$, $CB = 67$, and $CA = 41$. Paul stands at a point P on AB such that $CP = CA$. He aims to walk to touch side BC, then AC, then return to his start point. If the minimum distance he must walk is $a\sqrt{b}$, where a and b are positive integers with b being square-free, find $a + b$.

Problem 7. Let the maximum possible area of $ABCD$ be m, where A is the origin, B lies on the x-axis, D lies on the y-axis, and $BC + CD = 21$. If we can write $m = \frac{a+b\sqrt{c}}{d}$ where a, b, c, d are integers, a, b, d are relatively prime, and c is square-free, find $a + b + c + d$.

Problem 8. The Seattle Infinity Math City has two types of coins worth x cents and y cents, where x and y are positive integers satisfying $x > y$. There are exactly 2026 amounts that *cannot* be paid *exactly* by the residents of Seattle Infinity Math City. Let S be the sum of all possible $x + y$. Find S mod 1000.

Problem 9. Let x be the least number such that

$$x > \binom{3k}{k}^{\frac{1}{k}}$$

for all integers k. If the $x = \frac{m}{n}$ for relatively prime positive integers m and n, find mn.

Problem 10. A colored square is a unit square with each side colored blue or red such that there is either 1 or 3 blue sides. Call a colored square "blue-favored" if there are more blue than red sides and "red-favored" otherwise. A $m \times n$ rectangle composed of $mn \geq 1$ colored squares satisfies the property that if two squares are adjacent, their edge of adjacency is red in one square and blue in the other. If the outside border of the rectangle is entirely one color and the rectangle contains 2026 red-favored squares, let S be the sum of the possible values of $m + n$. Compute S mod 1000.

Problem 11. $AEBCD$ is a cyclic pentagon satisfying $\overline{AC} \perp \overline{BD}$ and $\overline{AB} \perp \overline{CE}$. If $EB = 25$, $BC = 56$, $CD = 39$, let the area of pentagon $AEBCD$ be X. Compute X mod 1000.

Problem 12. Let a *factorial representation* of an integer n be a non-negative integer sequence $(a_n)_{n\geq 1}$ such that $0 \leq a_k \leq k+1$ for all $k \geq 1$ and

$$\sum_{k=1}^{\infty} a_k k! = n.$$

How many integers n from 1 to 2025 inclusive have exactly two distinct factorial representations?

Problem 13. Let a and b be positive real numbers such that $b > a$, and let M be the maximum possible value of

$$\frac{a}{\sqrt{1+a^2}} + \frac{b-a}{\sqrt{(1+a^2)(1+b^2)}} + \frac{1}{\sqrt{1+b^2}}.$$

If the $M = \frac{m}{n}$ for relatively prime positive integers m and n, find $m + n$.

Problem 14. Two circles intersect at X and Y. The common tangent opposite X meets the two circles at P and Q. Suppose $[PXQY] = 200$, $PQ = 20$, and $XY = 21$. If the sum of the radii of the two circles can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n, find $m + n$.

Problem 15. A mosquito is flying in Edward's mosquito-trapping wire cage, a grid from $(0, 0, 0)$ to $(8, 8, 8)$, where every two lattice points are connected by a wire if and only if they are orthogonally adjacent. Every morning, the mosquito picks a new starting lattice point and flies along the wires until an ending lattice point. What is the minimum number of days it will take for the mosquito to fly along every single wire exactly once?

(Note that the mosquito does not need to start at the same point it ended the previous day.)